lar displacement of the body from the equilibrium position is ϕ. The symbol S represents the *torsional stiffness*, $[L/\phi]$, of the body.

Angular Velocity. — Figure 17 represents a body X that is vibrating with simple harmonic motion of rotation through an amplitude Φ about an axis normal to the plane of the diagram through the point O. The point p of a line op, fixed in the body, moves back and forth with simple harmonic motion of translation in the arc AB. The distance AB is shown as a straight line $A'B'$

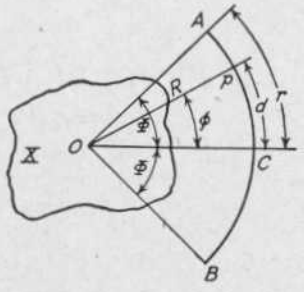

FIG. 17

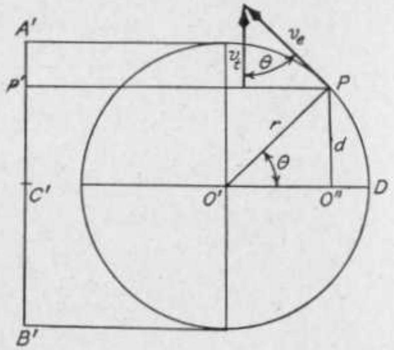

FIG. 18

in Fig. 18. The simple harmonic motion of translation along $A'B'$ is also the motion of the projection P' of a point P that moves with uniform speed in the circumference of a circle of diameter equal to $A'B'$ and in the same period as the simple harmonic motion of rotation of the body X.

Suppose that the point P moves with uniform speed v_e around the circle and that in time t after passing the position D, the radius $O'P$ has moved through an angle θ. At this instant, the component velocity in the direction $A'B'$, that is, the velocity of the projected point P', has the magnitude

$$v_t = v_e \cos\theta$$

In Fig. 18, the linear velocity of the point P' when at the equilibrium position is v_e, and the value at t seconds later when the body X has rotated through an angle ϕ, is v_t. Dividing each member of this equation by the distance R of the point p from the axis of vibration O,

$$\frac{v_t}{R} = \frac{v_e}{R}\cos\theta$$

or, (16):

$$w_t = w_e \cos\theta \qquad (38)$$

where w_e and w_t represent, respectively, the angular velocities of the body X when moving through the equilibrium position, and t seconds later.

In Fig. 18 the angle θ equals the product of the angular velocity of the line $O'P$ and the time t occupied in moving through this angle. If time T is occupied in making one entire revolution about O', then the angular velocity of $O'P$ has the magnitude $2\pi/T$. Hence, the angle

$$\theta = \frac{2\pi t}{T} \text{ radians} \qquad (39)$$

and (38) becomes

$$w_t = w_e \cos\frac{2\pi t}{T} \qquad (40)$$

In this equation time is reckoned from the equilibrium position. If it be reckoned from one end of the oscillation, that is $\pi/2$ radians from the equilibrium position, we would have

$$w_t' = w_e \cos\left(\frac{2\pi t}{T} - \frac{\pi}{2}\right) = w_e \sin\frac{2\pi t}{T} \qquad (41)$$

The maximum angular displacement from the equilibrium position is called the *amplitude* of the vibration. Another useful formula for the angular velocity is one involving the period T, angular amplitude Φ and angular displacement ϕ. From Fig. 18, the linear velocity of a body moving with simple harmonic motion of translation of period T and amplitude r at the instant when the displacement from the equilibrium position is d has the value

$$v_t[= v_e \cos\theta] = \frac{2\pi r}{T}\left(\frac{O'O''}{r}\right) = \frac{2\pi}{T}\sqrt{r^2 - d^2} \qquad (42)$$

From Fig. 17, (7), and (16)

$$r = \Phi R, d = \phi R \quad \text{and} \quad v_t = w_t R \qquad (43)$$

Substituting these values in (42) we find

$$w_t = \frac{2\pi}{T}\sqrt{\Phi^2 - \phi^2} \qquad (44)$$

At the equilibrium position, $\phi = 0$. When at this angular displacement, the velocity w_e has the magnitude, (44):

$$w_e = \frac{2\pi}{T}\Phi \text{ radians per sec.} \qquad (45)$$