The vibrations forced on the resonator lag behind the exciting force. When the frequencies of the exciter and the resonator are equal, the phase of the vibrations of the resonator are one-quarter period behind the vibrations of the exciter, and the amplitudes of the vibrations of the resonator are maximum. That is the resonator is vibrating through the maximum amplitude when the exciting force is zero. When the frequencies of the two vibrations are nearly equal, a small change in the frequency of either produces a considerable variation in the phase difference unless the damping of the resonant vibrations is great.*

Energy is alternately introduced into and withdrawn from the resonator by the exciter but the energy absorbed by the resonator is greater than the energy emitted. The energy absorbed is maximum when the frequency of the resonator is the same as that of the exciter.

Consider the case in which the natural frequency of vibration of $D$ and the frequency of the forced vibration impressed upon it are nearly equal. Suppose that one frequency is $n_1$ and the other is $n_2$. Then, during this time interval, the two vibrations are in phase $(n_1 - n_2)$ times. That is, during this time interval, the amplitude of vibration of the resultant motion will rise to a maximum and fall to a minimum $(n_1 - n_2)$ times. This is called the phenomenon of *beats*. If the pendulum $A$ is set into vibration when the periods of $A$ and $D$ are slightly different, then the amplitude of vibration of $D$ will increase to a maximum, fall to a minimum, and repeat. As the energy increasing the amplitude of the vibrations of $D$ comes from the pendulum $A$, the amplitude of the vibrations of the latter is minimum when the amplitude of $D$ is maximum. Energy of $D$ is now imparted to $A$ thereby increasing the amplitude of its vibrations. Often the frequency of a ship's roll is nearly equal to the frequency of the waves. When this occurs, the amplitudes of successive rolls will become larger and larger to a maximum, thereafter become smaller and smaller to a minimum, and then repeat. As the frequency with which the waves go under the ship depends upon the speed and course of the ship relative to the direction of the waves, the amplitude of roll can be altered by changing either the speed or the direction of the ship.

When the frequency of the forced vibration is much greater than the natural frequency of the body upon which it is impressed,

* Timoshenko, Vibration Problems in Engineering.



the effect upon the latter is very small. A seismograph for recording vibrations of the earth has a pendulum of such small frequency compared to the frequency of the tremors to be measured that it is not set into motion by rapid earthquake vibrations. The instrument records displacements of the earth relative to a pendulum of great moment of inertia and long period.

In this Article the case has been considered in which the periodic force varies harmonically, that is, in which the force is always directed toward the equilibrium position of the body and has a magnitude directly proportional to the displacement of the body from the equilibrium position. It should be noted, however, that if, instead of varying harmonically, the external force starts and stops suddenly but in a periodic manner, it will also set into resonant vibration a body of a free period that is a submultiple of the period of the external force. In this Article it has been assumed that the frequency of the exciter is unaffected by the resonator. The frequency of the exciter will be somewhat modified if either the stiffness or mass of the resonator be not less than that for the exciter.

**27. Damping of Vibrations.** — When linear vibrations are opposed by a force, or angular vibrations are opposed by a torque, the amplitude of each swing becomes less than the amplitude of the preceding swing. The successive diminution of the amplitudes of swing is called *damping*. If the body vibrating with simple harmonic motion of rotation be acted upon by a damping torque which at each instant is proportional to the angular speed of the body at that instant, then at all ordinary speeds the ratio of the amplitudes of any two successive swings will be constant. In this case the damping is constant.

Figures 27 to 31 represent a pendulous system consisting of a weighted bicycle wheel to which is rigidly attached a pair of connected tanks and partly filled with water or other liquid. The wheel is capable of rotation with negligible friction about a horizontal axle through the center. To produce a pendulum of long period, the moment of inertia with respect to the axis of oscillation is made large, and the torque about the axis of oscillation is made small, (50). In the present piece of apparatus, the moment of inertia is made large by replacing the ordinary tire of the bicycle wheel by a coil of wire of some twenty-five pounds. A small torque is produced by attaching a mass of a few ounces at a short distance below the axis of oscillation.