vessel of known weight F_g has been tilted from the upright position through the small angle ϕ by moving a load F across the deck through a distance x'. The center of gravity of the vessel with cargo is at G, the center of buoyancy at B, and the metacenter at M. The small angle through which the ship has been tilted is measured by means of a plumb-bob of length l hung at A. Represent by the symbol b the distance along the deck that the plumb-bob has moved when the vessel is tilted. Represent the metacentric height MG by the symbol H.

Now the moment of the tilting force equals the moment of the righting force. Taking moments about the metacenter and assuming that the angle of tilting is so small that x' and b are approximately equal to their horizontal projections,

$$Fx' = F_g x = F_g H \sin \phi = F_g H \frac{b}{l}$$

Whence, the metacentric height

$$H = \frac{Fx'l}{F_g b} \tag{97}$$

86. The Period of the Rolling Motion of a Ship. — By the period is meant the time of a complete back-and-forth oscillation. When a vessel has been rolled through an angle ϕ there is acting upon the vessel a righting torque, Fig. 105, having the value

$$L = F_g x = F_g H \sin \phi$$

where H is the metacentric height.

This equation shows that when the angle of roll ϕ is so small that $\sin \phi$ may be replaced by ϕ radians, then the restoring torque is directly proportional to ϕ. This is the law of simple harmonic motion of rotation (Art. 20). Consequently, if a vessel be rolled through a small angle from its equilibrium position, and if it be unacted upon by any torque except the righting torque, it will roll back and forth with a periodic motion that is approximately simple harmonic motion of rotation. On substituting in (37) the value of the torque $L = F_g H\phi$, we find that the period of such a simple harmonic motion of rotation is

$$T = 2\pi\sqrt{-\frac{K\phi}{L}} = 2\pi\sqrt{-\frac{K}{F_g H}} \tag{98}$$

where K is the moment of inertia of the vessel relative to the axis about which the rolling occurs. The negative sign indicates that

ϕ is measured in the direction opposite to L. Wind and frictional forces, and the forces due to the impacts of succeeding waves, produce such additional torques that the rolling motion is not simple harmonic and the value of the period cannot be computed with precision.

If, however, the rolling of a ship be assumed to be simple harmonic, then the H in (96) represents the metacentric height. In this case, if a ship carries a gyroscope of great angular momentum and with the spin-axle and the precession axis perpendicular to the axis of roll, then the period of roll is altered as it would be by a lengthening of H to $\left(H + \frac{h_s^2}{mgK_c}\right)$. Equation (98) shows that an increase of the metacentric height of a ship produces a decrease in the period of roll. Consequently, the rolling of a ship can be caused to be quicker by mounting on the ship a large gyro capable of spinning and precessing as above indicated.

87. Methods of Diminishing the Amplitude of Roll. — Roll increases stresses in the ship's structure and engines; it increases the effective area of cross-section of ship that must be pushed through the water, and consequently the fuel consumption; it decreases speed; it decreases the comfort of passengers and crew; it decreases the accuracy, range and rapidity of fire from naval vessels. These are ample reasons for the serious efforts made to diminish or suppress roll. Anti-roll devices are commonly called " ship stabilizers " because by their use the tilting of the ship from the equilibrium position by an applied torque is diminished.

The most obvious device is to attach long planks lengthwise on the outside of the hull below the water line. These so-called " bilge keels " decrease the roll but they also decrease the speed of the ship. To avoid the excessive friction through the water when the stabilizing effect of the bilge keels is not needed, it has been proposed to use fins that can be moved in and out of longitudinal slits through the hull of the ship.* When the ship is rolling, the fins would be protruded; when the ship is not rolling the fins would be withdrawn.

Frahm's anti-roll tanks (Art. 28) have been used to a considerable extent, although their mass and the difficulty in adjusting the size of the connecting passage to proper operation, have limited the use of the Frahm system.

* U. S. Patents. Thompson and Schein, No. 1475460, 1923; Motora, No. 1533328, 1925; Kéféli, No. 1751278, 1930.